Analysis on ultra-metric spaces and heat kernels

Alexander Grigoryan, Bielefeld University, Germany

Abstract
Given an ultra-metric space M with a reference measure and a cumulative distribution function on $[0, \infty)$, we construct a symmetric Markov semigroup whose generator L can be regarded as a non-local Laplace operator on such a space. Besides, it generates a jump process on M that is analogous to symmetric stable Levy processes in the Euclidean space. We obtain explicit formulas and estimates for the heat kernel and the Green function of L. In a particular case when M is the space of p-adic numbers (or its power), this construction recovers the Taibleson Laplacian. We apply this theory also to study of the Vladimirov Laplacian. Even in this well-established setting several of the results are new.